

Candidate Name

Candidate Number

Centre Name

Centre Number

Paper 1: Pure Mathematics 1 and Pure Mathematics 2

For Examination June 2023

(2 hours)

It is necessary to respond on the answer sheets provided alongside this question paper. Additionally, you must have a soft pencil (preferably of type B or HB), a clean eraser and a dark blue or black pen.

Instructions

- Answer all questions
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the spaces at the top of the page
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 9.81ms^{-2} .

Information

The Total for this paper is **80**

The number of marks for each question or part question is shown in brackets [].

2. A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points.

[5]

[illegible]

3. Find the term independent of x in each of the following expansions.

a) $\left(3x + \frac{2}{x^2}\right)^6$ [3]

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b) $\left(3x + \frac{2}{x^2}\right)^6 (1 - x^3)$ [3]

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4. The first term of a geometric progression and the first term of an arithmetic progression are both equal to a .

The third term of the geometric progression is equal to the second term of the arithmetic progression.
The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a .

[6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

5. (a) Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$.

[2]

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(b) The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}$$

Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied.

[4]

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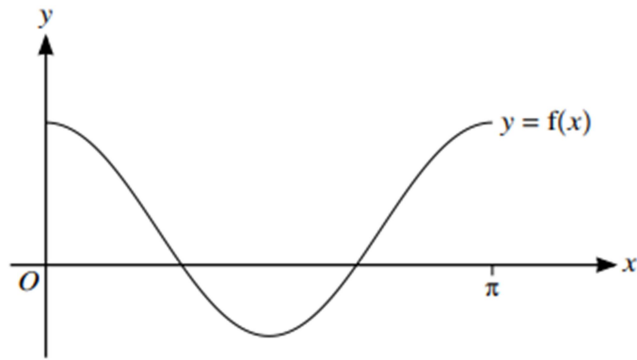
6. a) Show that $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} \equiv \frac{4}{5 \cos^2 \theta - 4}$.

[3]

[illegible]

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7.



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leq x \leq \pi$.

- (a) State the range of f . [2]

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A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x -axis is a tangent to the curve $y = g(x)$.

- (b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a \cos 2x + b$, where a and b are constants. [1]

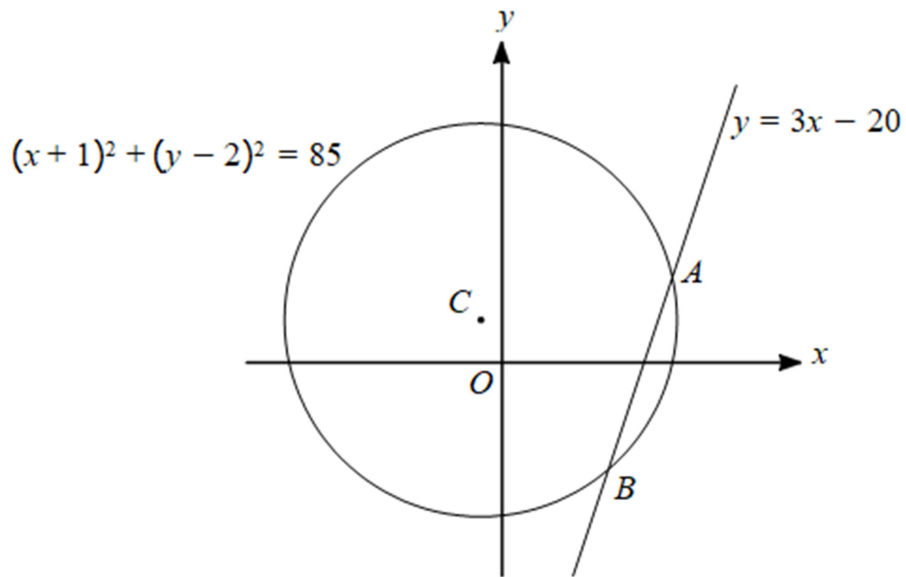
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8.



The circle with equation $(x+1)^2 + (y-2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

- (a) Find, by calculation, the coordinates of A and B.

[4]

[illegible]

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(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

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9. The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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[2]

[illegible]

10. The polynomial $p(x)$ is defined by

$$p(x) = 4x^3 + 16x^2 + 9x - 15.$$

(a) Find the quotient when $p(x)$ is divided by $(2x + 3)$, and show that the remainder is -6. [3]

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(b) Find $\int \frac{p(x)}{2x+3} dx$.

[2]

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11. The function f is given by $f(x) = 4 \cos^4 x + \cos^2 x - k$ for $0 \leq x \leq 2\pi$, where k is a constant.

(a) Given that $k=3$, find the exact solutions of the equation $f(x) = 0$. [5]

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