LRN International AS/A Level: Mathematics (7138)

Candidate Name	
Candidate Number	
Centre Name	
Centre Number	

Mathematics

Paper 1: Pure Mathematics 1 and Pure Mathematics 2 **For Examination December 2023**

(2 hours)

Instructions

- Answer all questions
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the spaces at the top of the page
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

Information

The Total for this paper is 90

The number of marks for each question or part question is shown in brackets.

1 Solve the equation $3x + 2 = \frac{2}{x - 1}$. [3]

2 The equation of a curve is such that $\frac{dy}{dx} = 12(\frac{1}{2}x - 1)^{-4}$. It is given that the curve passes through the point P 6, 4 .

(a) Find the equation of the tangent to the curve at P. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the equation of the curve. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

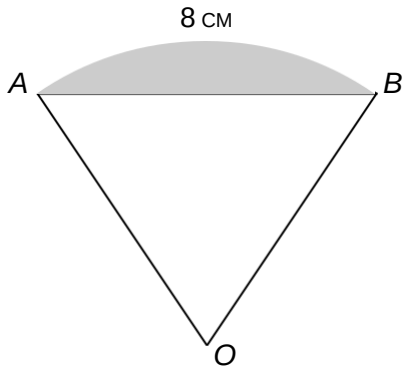
3 A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P, which has x-coordinate 9.

Find the y-coordinate of P. [5]

4 The coefficient of x^2 in the expansion of $\left(1+\frac{2}{p}x\right)^5 + (1+px)^6$ is 70.

Find the possible values of the constant p. [6]

5



The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the area of the shaded segment. [2]

6 (a) Show that the equation

$$\frac{1}{\sin + \cos} + \frac{1}{\sin - \cos} = 1$$

may be expressed in the form $a \sin^2 + b \sin + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$ for $0 \leq \theta \leq 360$. [3]

7 A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$ where p is a constant.

(a) Find f '(x) and hence determine whether f is an increasing function, decreasing function or neither. [3]

9 Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx-d}$, where a, b, c, d are integers. [4]

(b) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$ [1]

10 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$f(x) = x^2 - 4x + 9$,

$g(x) = 2x^2 + 4x + 12$,

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Express g (x) in the form kf (x + h), where k and h are integers. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Describe fully the two transformations that have been combined to transform the graph of y = f (x)

to the graph of y = g (x) . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11 Given that $\ln(2x + 1) - \ln(x - 3) = 2$,

find x in terms of e. [4]

12 The polynomial p x is defined by

$$\text{p}\,x = x^3 + ax^2 + bx + 16,$$

where a and b are constants. It is given that $x + 2$ is a factor of p x and that the remainder is 72 when p x is divided by $x - 2$.

Find the values of a and b. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

13 i) Solve the equation $|2x - 5| = |x + 6|$. [3]

ii) Hence find the value of y such that $|2^{1-y} - 5| = |2^{-y} + 6|$. Give your answer to 3 significant figures. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

14 The sequence of values given by the iterative formula $x_{n+1} = \frac{6 + 8x_n}{8 + x_n^2}$ with initial value $x_1 = 2$ converges to α.

a) Use the iterative formula to find the value of α correct to 4 significant figures. Give the result of each iteration to 6 significant figures.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

b) State an equation satisfied by α and hence determine the exact value of α.

[2]

.....

………

………

………

………

………

………

………

………

………

………

………

………

………

………

………

………

………

………

………

………

15 It is given that $3 \sin 2\theta = \cos \theta$ Where θ is an angle such that $0 < \theta < 90^{o}$.

(a) Find the exact value of $\sin \theta$. [2]

……

……

……

.....

.....

.....

.....

(b) Find the exact value of sec θ. [2]

.....

.....

.....

.....

.....

.....

.....

(c) Find the exact value of cos 2θ. [2]

.....

.....

.....

.....

.....

.....

.....

16 A curve has equation y = f(x) where f(x) = $\frac{4x^3 + 8x - 4}{2x - 1}$.

(a) Find an expression for dy/dx and hence find the coordinates of each of the stationary points of the curve y= f(x). [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Divide $4x^3 + 8x - 4$ by $(2x - 1)$, and hence find $\int f(x)\,dx$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of Paper