

Candidate Name

Candidate Number

Centre Name

Centre Number

Paper 3: Pure Mathematics 1, Pure Mathematics 2 and Pure Mathematics 3

For Examination December 2023

Time: 2 hours

Instructions

- Answer all questions
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the spaces at the top of the page
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

Information

- The Total for this paper is 90
- The number of marks for each question or part question is shown in brackets.

Question 1

Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answers correct to 3 decimal places. [4]

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Question 2

Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$.

[5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 3

$$\text{Let } f(\theta) = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}.$$

i) Show that $f(\theta) = \tan(\theta)$

[3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

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Question 3 continued

ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} f(\theta) \, d\theta = \frac{1}{2} \ln \frac{3}{2}.$ [4]

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Question 4

The curve $y = \sin\left(x + \frac{1}{3}\pi\right) \cos x$ has two stationary points in the interval $0 \leq x \leq \pi$.

- i) Find $\frac{dy}{dx}$ [2]

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- [2]

[illegible]

[3]

A series of horizontal dotted lines spanning the width of the page, intended as a guide for handwriting practice.

Question 5

The complex number u is defined by

$$u = \frac{4i}{1 - (\sqrt{3}i)}.$$

- (i) Express u in the form $x + iy$, where x and y are real and exact.

[3]

This image shows a full page of primary-ruled paper. It features multiple horizontal rows, each defined by two parallel dashed lines. The rows are evenly spaced across the entire page, providing a guide for handwriting practice. There are no margins, text, or other markings present.

(ii) Find the exact modulus and argument of u .

[2]

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Question 6

Prove, using algebra, that

$n(n^2 + 5)$ is even for all $n \in \mathbb{N}$

[4]

[illegible]

Question 7

The function f is defined by

$$f(x) = \frac{8x+5}{2x+3} \quad x > -\frac{3}{2}$$

- (a) Find $f^{-1}\left(\frac{3}{2}\right)$ [2]

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- (b) Show that

$$f(x) = A + \frac{B}{2x+3} \quad \text{where A and B are constants to be found.} \quad [2]$$

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The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leq x \leq 4$$

- (c) State the range of g^{-1} [1]

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- (d) Find the range of $f \circ g^{-1}$ [3]

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Question 8

Let $f(x) = \frac{2x(5-x)}{(3+x)(1-x)^2}$

- (i) Express $f(x)$ in partial fractions. [5]

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[5]

[illegible]

Question 9

The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

- (i) The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l . [5]

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- (ii) It is given that l lies in the plane with equation $ax + by + 2z = 13$, where a and b are constants. Find the values of a and b . [6]

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Question 10

The equation of a curve is

$$Y = 3 \cos 2x + 7 \sin x + 2.$$

Find the x-coordinates of the stationary points in the interval $0 \leq x \leq \pi$. Give each answer correct to 3 significant figures.

[7]

[illegible]

Question 11

(a) Find $\int (4 + \tan^2 2x) dx$.

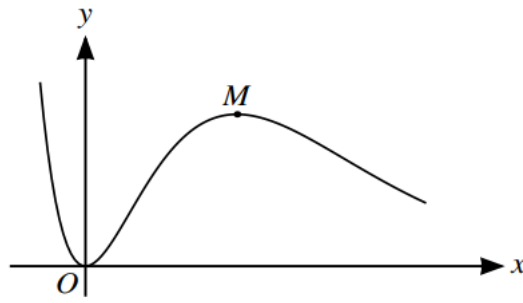
[3]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dashed lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} dx$.

[5]

Question 12



The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M .

- i) Show that the X-coordinate of M is 2.

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- ii) Find the exact value of $\int_0^2 x^2 e^{2-x} dx$.

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Question 15

Prove by contradiction that $\sqrt{2}$ is irrational. [6]

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End of Paper