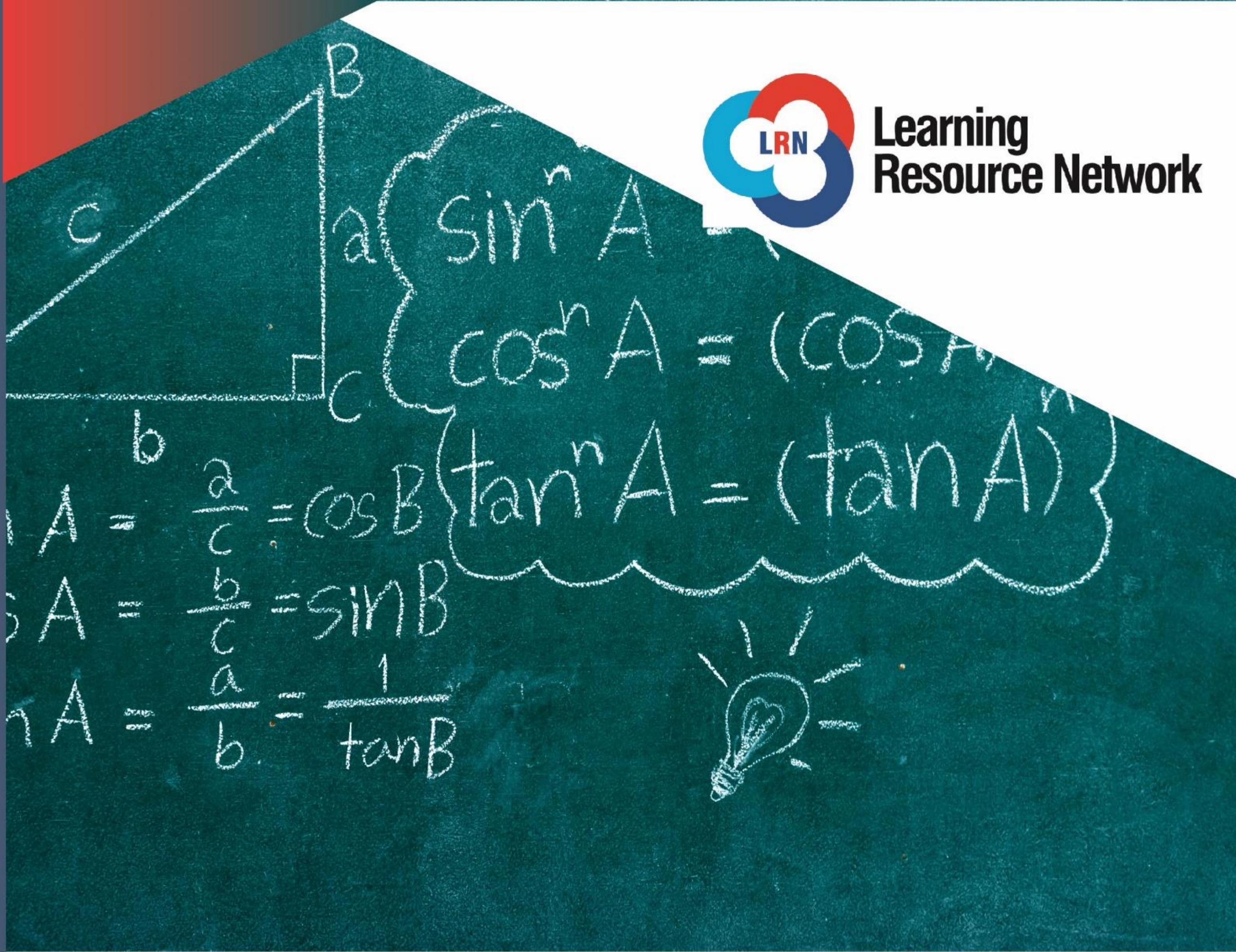
Learning Resource Network



INTERNATIONAL AS AND A-LEVEL MATHEMATICS (7138)

SPECIFICATION

For exams January, May and November onwards
For teaching from September 2021 onwards

THE QUEEN'S AWARDS FOR ENTERPRISE: INTERNATIONAL TRADE 2020

Contents

BACKGROUND TO LRN

Learning Resource Network (LRN) is a recognised Awarding Organisation that offers a range of qualifications to candidates, educational institutes, training providers, schools and employers.

LRN is recognised for its high quality qualifications that enable candidates to progress to other areas of study and employment in their designated fields.

In producing its qualifications, LRN uses the experience and expertise of academics, professionals working in the pertinent industries and assessment practitioners with a wealth of best practice and knowledge of validation, verification, delivery and assessment.

ACCOLADES

Queen’s Award

In April 2020, LRN received the Queen’s Award for Enterprise for International Trade. LRN is one of 220 organisations in the UK to be recognised with this prestigious accolade. This was in recognition of the expansion LRN brought to the overseas qualification market.

MANAGEMENT SYSTEMS

LRN has been awarded international accreditation as part of its quality controls, policies, systems and overall approach to its management systems. These awards are externally validated by the British Assessment Bureau. LRN has achieved accreditation in the form of ISO 9001: Quality Management Systems, ISO 14001: Environment Management Systems and ISO 27001: Information Security Management Systems.

CUSTOMER SERVICE EXCELLENCE

LRN has achieved the prestigious award of Customer Service Excellence. This is in recognition of its customer service practices, approach to managing and dealing with UK and Overseas customer needs, including the diverse needs of its centres.

LRN was the first UK Awarding Organisation to achieve Customer Service Excellence.
Following reaccreditation in 2019, LRN received an award for Customer Service Excellence: Compliance Plus, demonstrating that LRN went above and beyond the delivery of its customer service principles.

British Assessment Bureau UKAS MANAGEMENT SYSTEMS 8289 ISO 9001 QUALITY MANAGEMENT

British Assessment Bureau UKAS MANAGEMENT SYSTEMS 8289 ISO 27001 INFORMATION SECURITY MANAGEMENT

British Assessment Bureau UKAS MANAGEMENT SYSTEMS 8289 ISO 14001 ENVIRONMENTAL MANAGEMENT

CUSTOMER SERVICE EXCELLENCE® CSE

INTRODUCTION

This specification provides an overview to the LRN International AS & A Level Mathematics[1]. This document is suitable for various users, including candidates, centres, administrators, employers, parents/guardians, teachers (and other educational based staff) and examiners. The specification outlines the key features and administrative procedures required for this international qualification.

OBJECTIVE

The LRN International AS & A Level Mathematics is designed to enable international candidates to demonstrate their ability, in theoretical terms across a range of: Pure Mathematics, Mechanics and Probability and Statistics.

MODE OF DELIVERY

This qualification has been constructed to be delivered within centres. Centres will need to demonstrate to LRN, through the centre recognition processes, that they have the resources, facilities and competence to deliver. However, centres must be able to demonstrate, in line with LRN's criteria, that they have the means, capability, capacity and resources (including suitably qualified centre staff) to deliver by the method chosen by the centre.

PROGRESSION

The LRN International AS & A Level Mathematics has been designed to reflect the wide variation in candidates' origins, levels of education and career aims. Progression opportunities may, therefore, take a variety of paths. Depending on the level of qualification achieved, it may be appropriate for the candidate to progress to:

1. Similar level 3 qualification in Mathematics;
2. LRN Level 3 Diploma in Pre-U Foundation Studies;
3. A higher level of any qualification – e.g.; HNC/HND or Degree'
4. Vocationally Related Qualifications

[1] LRN International AS/A Level are globally recognised qualifications designed specifically for international candidates and are available outside the United Kingdom. Candidates based in England refer to the Ofqual register.

QUALIFICATION OVERVIEW

Number	Subject Content	LRN International AS Level	LRN International A Level	AO	Exam
1	Pure Mathematics 1	√	√	1, 2 and 3	Combination of written exam papers (externally set and marked). **AS Level** **Paper 1:** structured questions based on the Pure Mathematics (1 & 2) subject content. Duration:2 hours Weighting: 66% **Paper 2:** structured questions based on the Mechanics 1. Duration: 1 hour 15 minutes Weighting: 34% **A Level** **Paper 1:** structured questions based on the Pure Mathematics (1 & 2) subject content. Duration:2 hours Weighting: 33.33% **Paper 3:** structured questions based on the Pure Mathematics (1,2 & 3) subject content. Duration:2 hours Weighting: 33.33%
2	Pure Mathematics 2	√	√	1, 2 and 3	
3	Pure Mathematics 3	-	√	1, 2 and 3	
4	Mechanics 1	√	√	1, 2 and 3	
5	Mechanics 2	-	√	1, 2 and 3	
6	Probability and Statistics	-	√	1, 2 and 3	

					Paper 4: structured questions based on the Mechanics (1 & 2) and Probability and Statistics. Duration: 2 hours Weighting: 33.33%

BREAKDOWN OF ASSESSMENT OBJECTIVES

AO1 - Use and apply standard techniques:

- select and correctly carry out routine procedures
- accurately recall facts, terminology and definitions

AO2 – Reason, interpret and communicate mathematically:

- construct rigorous mathematical arguments (including proofs)
- make deductions and inferences
- assess the validity of mathematical arguments
- explain their reasoning
- use mathematical language and notation correctly

AO3 – Solve problems within mathematics and in other contexts:

- translate problems in mathematical and non-mathematical contexts into mathematical processes
- interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations
- translate situations in context into mathematical models
- use mathematical models
- evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them

ASSESSMENT

The assessment for this qualification consists of (i) written exam papers, set and marked by the LRN.

Assessment objectives (AOs)	Weighting			
	Paper 1	Paper 2	Paper 3	Paper 4
AO1	45%	45%	40%	30%
AO2	35%	35%	35%	40%
AO3	20%	20%	25%	30%

GUIDED LEARNING HOURS (GLH)

The LRN International AS Level guided learning hours (GLH) are 180 and 360 guided learning hours for LRN International A Level. Please note the hours stated are indicative.

ENTRIES CODES

One entry per qualification is sufficient and will cover all the question papers including certification.

PRIVATE CANDIDATES

Centres are advised that private candidates are only to be enrolled with prior agreement and confirmation from LRN.

GRADING

The LRN International A Level will be graded on a six-point scale: A*, A, B, C, D and E and LRN International AS Level will be graded on a five-point scale: A, B, C, D and E Candidates who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

RESULTS

Exam series are in:

- January (results released in March)
- June (results released in August)
- November (results released in January)

RE-TAKES

Whereas candidates can re-take each paper as often as they wish, within the shelf-life of the specification.

CUSTOMER SERVICE STATEMENT

Learning Resource Network (LRN) is committed to ensuring all customers are dealt with promptly and in a professional and helpful manner. In order to guarantee this, we commit to ensuring the following in our day-to-day interactions with candidates, assessment centres and our stakeholder network:

- All customers will be treated equally and with respect.
- All customer information will only be used in a way which has been agreed in advance, unless we are informed of something that places them or others at risk of harm.
- All customers will be treated by staff in a professional manner.

LRN has arrangements in place to provide a telephone and e-mail helpdesk which will be staffed from 09:00 to 17:00 from Monday to Friday. Furthermore, it will respond to each e-mail, letter, or telephone message it receives regarding feedback on its qualifications, centre approvals process or other matters relating to its products and/or services. The timetable for responding is as follows:

- E-mail: 5 working days
- Letter: 5 working days
- Telephone message: 5 working days

DIVERSITY AND EQUALITY

Learning Resource Network (LRN) is committed to ensuring fair and equal access to its qualifications, examinations and support materials. Our Diversity and Equality policy seeks to eliminate unjustifiable discrimination, harassment and/or victimisation and to advance equality of opportunity, thereby ensuring all candidates are treated fairly, in accordance with the protected characteristics of the Equality Act 2010. Specifically, we comply fully with the requirements laid out in the Equality Act 2010. In addition, and within the constraints of this policy, LRN will have due regard for the General data Protection Regulations (GDPR) in the retention of information which is unnecessary.

1	Pure Mathematics 1		
Aim Complex mathematics builds on the foundation of simpler mathematics. The aim of this subject content is to gain an understanding and familiarity with the basics of mathematical notation, algebraic manipulation, functions and graphs, trigonometry and calculus.			
Learning Outcomes - The learner will:		**Assessment Criteria - The learner can:**	
1	Understand algebraic expressions.	1.1	**Expand** brackets, collect like terms, factorise expressions, and rearrange equations to simplify or extract information (e.g., roots of polynomials).
		1.2	**Know and use** the notation and meaning of positive and negative rational indices.
		1.3	**Use** the index laws to simplify expressions: $x^m \times x^n \equiv x^{m+n}$ $x^m \div x^n \equiv x^{m-n}$ $(x^m)^n \equiv x^{mn}$ $(xy)^n \equiv x^n y^n$
		1.4	**Know and understand** that a 'Surd' is an expression containing unresolved (non-rational) roots.
		1.5	**Use** the surd laws to simplify expressions (special cases of the index laws): $\sqrt[n]{xy} = \sqrt[n]{x} \times \sqrt[n]{y}$ $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$
		1.6	**Know and understand** that $(x+y)$ and $(x-y)$ are 'Conjugate Pairs', and that $(x+y)(x-y) = x^2 + y^2$.
		1.7	**Rationalise** the denominator of a fraction by multiplication of the numerator and the denominator by the conjugate of the denominator, e.g. for surds $\frac{1}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$ and $\frac{1}{\sqrt{x+y}} \times \frac{\sqrt{x-y}}{\sqrt{x-y}} = \frac{\sqrt{x-y}}{x^2+y^2}$.
		1.8	**Know** that irrational numbers should be left on the simplest surd form and not converted to a decimal.
2	Understand quadratic equations.	2.1	**Know** that quadratic equations are of the form $ax^2 + bx + c = 0$ for $a \neq 0$, where x is a variable or a function and a, b, and c are real constants.
		2.2	**Factorise** quadratic equations and extract solutions from the factors.
		2.3	**Know and understand** that quadratic equations have up to two real solutions, that these solutions are called 'Roots', and that if only one real root exists then it is called a 'Repeated Root'.

		2.4	"**Complete** the Square" for a quadratic equation by rearranging it into the form $p(x+q)^2+r=0$ and extract the solutions $x=-q\pm\sqrt{-\frac{r}{p}}$.
		2.5	**Know and use** the fact that if quadratic equation is in the completed square form then the graph of the equation has a turning point at (-q,r).
		2.6	**Know and use** the 'Quadratic Formula', $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, and extract the number of real roots from the 'Discriminant', b^2-4ac.
		2.7	**Solve** quadratic equations and quadratic inequalities in one unknown using factorisation, completing the square or the quadratic formula.
		2.8	**Recognise and solve equations** which are quadratic in a function, e.g. $x^2+2=0$ where $x=t^2$, $x=\sqrt{t}$, or $x=\sin(t)$.
		2.9	**Solve** simultaneous equations where one is quadratic and the other linear by substitution.
		2.10	**Sketch** graphs of quadratic equations and identify the turning point and any axis intercepts.
3	Understand functions.	3.1	**Know and use** the symbols for, and definitions of, the 'Real Numbers' $\mathbb{R}$, 'Integers' $\mathbb{Z}$, 'Natural Numbers' $\mathbb{N}$, and 'Rational Numbers' $\mathbb{Q}$.
		3.2	**Know and understand** that a 'Map' is a transform that takes numbers in one set as inputs and returns numbers from another set.
		3.3	**Know and understand** that maps can be 'One-to-One', 'One-to-Many or many-to-one, and that they are either 'Onto' or 'Not Onto'. {The terms injective, surjective and bijective are not needed, but correct use will be credited}
		3.4	**Know and understand** that the 'Domain' of a map is the set that the map transforms from, and that the 'Range' of a map is the set that the map transforms into.
		3.5	**Know and understand** that a 'Function' is a map that is one-to-one or one-to-many.
		3.6	**Identify** the properties of a map, including domain and range, and justify whether or not it is a function.
		3.7	**Know and use** the notation for the 'Composition' of two functions, $fg(x)=f(g(x))$, and how to apply the composition.

		3.8	**Describe and sketch** transformations of simple functions. For $y = f(x)$: $y = f(x) + a$ is a 'Translation' by $+a$ along the y-axis $y = f(x + a)$ is a 'Translation' by $-a$ along the x-axis $y = af(x)$ is a 'Stretch' by the 'Scale Factor' a in the y-direction $y = f(ax)$ is a 'Stretch' by the 'Scale Factor' $\frac{1}{a}$ in the x-direction $y = -f(x)$ is a 'Reflection' in the x-axis $y = f(-x)$ is a 'Reflection' in the x-axis
		3.9	**Know and use** the properties of the 'Inverse' of a one-to-one function, and use the notation $f^{-1}(x)$ is the inverse of $f(x)$. $ff^{-1}(x) = f^{-1}f(x) = x$ The graph of $y = f^{-1}(x)$ is the graph of $y = f(x)$ reflected in the line $y = x$ The domain of $f(x)$ is the range of $f^{-1}(x)$ The range of $f(x)$ is the domain of $f^{-1}(x)$
		3.10	**Find** **Know and understand** how to find the inverse of a function, determine its domain and range, and sketch the function, the inverse and the line $y = x$ on a graph.
4	Understand coordinate geometry.	4.1	**Know and understand** that the general form of a straight line is $y = mx + c$ where m is the gradient of the line, and c is the y-axis intercept.
		4.2	**Recognise** the alternate forms for the equation of a straight line, $y - y_1 = m(x - x_1)$, and $ax + by + c = 0$.
		4.3	**Determine** if three points are 'Colinear' by finding a straight line that passes through all three points.
		4.4	**Know and understand** how to find the equation of a straight line given two points on the line: If (x_1, y_1) and (x_2, y_2) are on the line then $m = \frac{y_2 - y_1}{x_2 - x_1}$ and $c = y_1 - mx_1$.

		4.5	**Know and understand** how to find the equation of a straight line given a point and the gradient: If (x_1, y_1) is a point on a line with gradient m then $y - y_1 = m(x - x_1)$.
		4.6	**Show** that two straight lines are parallel, perpendicular or neither.
		4.7	**Know and understand** how to find the distance along a straight line between two points, the mid-point between two points on a straight line, and the area bounded by $y = mx + c, y = 0, x = a$ and $x = b$.
		4.8	**Know and understand** how to find the mid-point between two points on a straight line.
		4.9	**Know and understand** how to find the area bounded by $y = mx + c, y = 0, x = a$ and $x = b$.
		4.10	**Know and understand** how to find the 'Point of Intersection' between two straight lines.
		4.11	**Know and understand** how to find the 'Perpendicular Bisector' of a line segment.
		4.12	**Know and use** the general form of the equation for a circle with its centre at (a, b), and radius r, $(x - a)^2 + (y - b)^2 = r^2$.
		4.13	**Recognise** the alternate forms for the equation of a circle, $x^2 + y^2 + 2\alpha x + 2\beta y + \gamma = 0$, with centre $(-\alpha, -\beta)$ and radius $\sqrt{(\alpha^2 + \beta^2 - \gamma)}$.
		4.14	**Know and understand** how to find the points of intersection of a circle and a straight line.
		4.15	**Know and understand** how to find the equation of a tangent to a circle at a given point.
		4.16	**Know and understand** how to find the equation of the perpendicular bisector of a chord of a circle.
		4.17	**Know and understand** how to find the points of intersection of a quadratic and a straight line, and hence find the solutions for the related quadratic equation.

5	Understand angles and trigonometry.	5.1	**Know and understand** the definition of a 'Radian' and convert between radians and degrees.
		5.2	**Know and use** the exact conversions between radians an degrees for 30º, 45º, 60º, 90º, 180º and 360º.
		5.3	**Solve** problems involving the 'Arc Length', $s = r\theta$, and 'Sector Area', $A = \frac{1}{2}r^2\theta$, of a circle.
		5.4	**Sketch and use** graphs of the 'Sine', 'Cosine' and 'Tangent' for angles in degrees or radians.
		5.5	**Sketch and use** graphs of simple transformations of sin, cos and tan.
		5.6	**Know and use** the exact values of sin, cos and tan for the angles 30º, 45º, 60º and other related angles.
		5.7	**Use** the inverse functions $\sin^{-1}x$, $\cos^{-1}x$, and $\tan^{-1}x$ to find unknown angles in the principle range, and the properties of sin, cos and tan to calculate alternative solution. Justify the chosen solution.
		5.8	**Use** the 'Trigonometric Identities' $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta \equiv 1$ to prove identities and simplify expressions.
		5.9	**Know and understand** how to find all the solutions of equations involving trigonometric functions within a specified interval.
		5.10	**Use** the trigonometric functions to find the unknown sides or angles of right-angled triangles.
		5.11	**Use** the 'Sine Rule', $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and the 'Cosine Rule', $a^2 = b^2 + c^2 - 2bc\cos A$ to find unknown sides or angles of general triangles.
		5.12	**Know and understand** how to find the area of a triangle using $A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}ab\sin C$.
		5.13	**Know and understand** how to find the area of a 'Segment' of a circle.
6	Understand differentiation.	6.1	**Understand** that 'Differentiation' comes from finding the gradient of a chord on a curve as the length of the chord goes to zero. {Differentiating from first principles and derivation of the gradient function will not be examined.}
		6.2	**Know and understand** that the 'Derivative' of a function f at a point is the gradient of f at that point, and that the derivative of f is the function that describes the gradient of f at every point.
		6.3	**Recognise** the notation for taking limits, $\lim_{h \to 0}$.

		6.4	**Know and understand** that $\frac{d}{dx}$ is the operator that produces the derivative with respect to x, and that it can be written in ways $\frac{d}{dx}f(x) = \frac{dy}{dx} = f'(x) = y'$, where $y = f(x)$.
		6.5	**Know and understand** how to find the derivative of x^n for $n \neq 0$. {Powers of x may need to be consolidated first}
		6.6	**Know and understand** how to find the derivative of functions with multiple terms by applying differentiation to each term individually, If $y = f(x) + g(x)$ then $y' = f'(x) + g'(x)$.
		6.7	**Know and understand** how to find the derivative of constant multiples of functions or polynomials using $\frac{d}{dx}\left(Af(x)\right) = A\frac{d}{dx}f(x) = Af'(x)$.
		6.8	**Know and understand** how to find the equations for the tangent $y - f(a) = f'(a)(x-a)$ and normal $y - f(a) = -\frac{1}{f'(a)}(x-a)$ for a function $f(x)$ at the point $x = a$.
		6.9	**Know and understand** that the second derivative is the derivative of the derivative, and that it produces the function that is the rate of change of the gradient function.
		6.10	**Know and understand** that $\frac{d^2}{dx^2}$ is the operator that produces the second derivative with respect to x, and that it can be written in equivalent ways $\frac{d^2}{dx^2}f(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = f''(x) = y''$, where $y = f(x)$.
7	Understand integration.	7.1	**Know and understand** that 'Integration' is the inverse of differentiation, that the 'Integrand' is the function to be integrated, and that the 'Constant of Integration' accounts for the derivatives of constants equalling zero.
		7.2	**Know and understand** that the 'Indefinite Integral' of $f'(x)$ is the set of functions that differentiate to $f'(x)$, and use the integral notation $\int f'(x)dx = f(x) + c$.
		7.3	**Know and understand** how to find the integral of x^n for $n \neq -1$. {Powers of x may need to be consolidated first}
		7.4	**Know and understand** how to find the integral of functions with multiple terms by applying integration to each term individually, If $y = f'(x) + g'(x)$ then $\int y\,dx = \int f'(x)dx + \int g'(x)dx = f(x) + g(x) + c$.
		7.5	**Know and understand** how to find the integral of polynomial functions.
		7.6	**Know and understand** how to find the value of c given a point that the integral passes through.

2	Pure Mathematics 2		
Aim Here we build on the foundations laid in Pure Mathematics 1. The aim of this subject content is to continue on with more advanced algebraic techniques, trigonometry and calculus, while introducing exponentials and logarithms, sequences and series, and numerical methods for solving equations. Note: There is the expectation that the content from Pure Mathematics 1 is known and it will be tested as part of this unit without being included below.			
Learning Outcomes - The learner will:		**Assessment Criteria - The learner can:**	
1	Understand algebra.	1.1	**Know and understand** the definition of the ‘Modulus (or Absolute Value) Function’, $\lvert f(x)\rvert$, where $f(x)$ is the ‘Argument’ of the modulus.
		1.2	**Sketch** graphs of the form $y = \lvert ax + b\rvert$, and $y = f(\lvert x\rvert)$ for non-linear functions.
		1.3	**Know and understand** how to find the solutions to modulus equations.
		1.4	**Use** the relationships $\lvert a\rvert = \lvert b\rvert \iff a^2 = b^2$ and $\lvert x - a\rvert < b \iff a - b < x < a + b$ to solve inequalities and equations.
		1.5	**Perform** algebraic division of polynomials (maximum degree 4) using long division and factorisation, identifying the ‘Quotient’ and ‘Remainder’.
		1.6	**State** the ‘Factor Theorem’ - For a polynomial $p(x)$: If $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$ If $(x - a)$ is a factor of $p(x)$ then $p(a) = 0$ If $p\left(\frac{a}{b}\right) = 0$ then $(bx - a)$ is a factor of $p(x)$ If $(bx - a)$ is a factor of $p(x)$ then $p\left(\frac{a}{b}\right) = 0$
		1.7	**Use** the factor theorem to factorise polynomials.
		1.8	**State** the ‘Remainder Theorem’ – If a polynomial $p(x)$ is divided by $(ax - b)$ then the remainder is $p(\frac{b}{a})$.
		1.9	**Solve** polynomial equations and equations involving quotients of polynomials using the factor and remainder theorems, and algebraic division.
2	Understand logarithms and exponentials.	2.1	**Know and use** the definitions of ‘Logarithms’ and ‘Exponentials’, and the link between them as inverse functions.
		2.2	**Know and use** the link between the ‘Natural Logarithm’ and the constant e.

		2.3	**Know and use** the 'Laws of Logarithms: $\log_b x + \log_b y = \log_b xy$ {'Multiplication Law'} $\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$ {'Division Law'} $\log_b\left(x^k\right) = k \log_a x$ {'Power Law'} and the special cases: $\log_b \left(\frac{1}{x}\right) = -\log_b x$ $\log_b b = 1$ where $b > 0$ and $b \neq 1$ $\log_b 1 = 0$ where $b > 0$ and $b \neq 1$
		2.4	**Know and use** $\log_a x = \frac{\log_b x}{\log_b a}$ and $\log_a b = \frac{1}{\log_b a}$ to change the 'Base' of a logarithm.
		2.5	**Sketch** the graphs of $y = e^{kx}$ for positive and negative k, and $\ln x$.
		2.6	**Solve** equations and inequalities where the unknown is in an exponent.
		2.7	**Use** logarithms to transform exponential equations into a linear form and solve by considering the gradient and the intercept.
3	Understand sequences and series.	3.1	**Use** the Sigma notation, $\sum_{i=a}^{b} f_i(x)$, to represent the sum of a series of terms.
		3.2	**Use** 'Pascal's Triangle' to expand $(a+b)^n$.
		3.3	**Know and understand** that '*n* Factorial' is the product of every integer from 1 to *n*, that *n* factorial is written $n!$, and that by definition $0! = 1$.
		3.4	**Know and understand** that the number of ways of choosing *r* items from a set of *n* items, *n* choose *r*, is written ${}^nC_r$ or $\binom{n}{r}$, and is given by ${}^nC_r = \frac{n!}{r!(n-r)!}$.
		3.5	**Calculate** the *r*th entry in the *n*th row of Pascal's triangle, which is given by ${}^{n-1}C_{r-1}$.
		3.6	**Use** the 'Binomial Expansion', $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$, to expand powers of brackets for $n \leq 5$.
		3.7	**Approximate** $(a+b)^n$ for large n, and estimate large powers of a number $0 < m < 1$ by taking the first few terms of the binomial expansion.
		3.8	**Recognise** 'Arithmetic Sequences' and 'Geometric Sequences', calculate the *n*th term and the sum of the first *n* terms.

		3.9	**Determine** if a geometric series is 'Convergent' and calculate the sum to infinity is the series is convergent.
4	Understand trigonometry.	4.1	**Know and understand** how to find the solutions for equations of the form $\sin(n\theta) = a$, $\cos(n\theta) = a$, and $\tan(n\theta) = b$, where $-1 \leq a \leq 1$ and $-\infty \leq b \leq \infty$, in a given interval.
		4.2	**Know and understand** how to find the solutions for equations of the form $\sin(\theta + \alpha) = a$, $\cos(\theta + \alpha) = a$, and $\tan(\theta + \alpha) = b$, where $-1 \leq a \leq 1$ and $-\infty \leq b \leq \infty$, in a given interval.
		4.3	**Know and use** the definitions of the 'Reciprocal Trigonometric Functions', 'Secant', 'Cosecant', and 'Cotangent', and the values for which they are undefined.
		4.4	**Sketch and use** graphs of the sec, cosec and cot for angles in degrees or radians.
		4.5	**Use** the 'Addition Formulae' to solve equations and prove identities: $\sin A + \sin B \equiv 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\sin A - \sin B \equiv 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\cos A + \cos B \equiv 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\cos A - \cos B \equiv 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
		4.6	**Use** the 'Compound-Angle Formulae' to solve equations and prove identities: $\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ where $(A \pm B) \neq \left(n + \frac{1}{2}\right)\pi$
		4.7	**Use** the addition formulae to produce the 'Double-Angle Formulae'.
		4.8	**Express** arbitrary sums of a sin and a cos function in terms of a single trig. Function: $a\sin x \pm b\cos x = R\sin(x \pm \alpha)$ $a\cos x \pm b\sin x = R\cos(x \mp \alpha)$ where $R > 0$, $0 < \alpha < \frac{\pi}{2}$, $R\cos\alpha = a$, $R\sin\alpha = b$, and $R = \sqrt{a^2 + b^2}$.
		4.9	**Use** 'Reciprocal Trigonometric Identities' to solve equations and prove identities: $\sec^2\theta \equiv 1 + \tan^2\theta$ $\mathrm{cosec}^2\theta \equiv 1 + \cot^2\theta$
5	Understand differentiation.	5.1	**Know and understand** that a function $f(x)$ is increasing in the interval $[a, b]$ if $f'(x) \geq 0$ for all values of x such that $a < x < b$, and strictly increasing if $f(x) > 0$.
		5.2	**Know and understand** that a function $f(x)$ is decreasing in the interval $[a, b]$ if $f'(x) \leq 0$ for all values of x such that $a < x < b$, and strictly increasing if $f(x) < 0$.

		5.3	**Know and understand** that a function $f(x)$ is unchanging in the interval $[a, b]$ if $f'(x) = 0$ for all values of x such that $a < x < b$.
		5.4	**Determine** whether a function is increasing, decreasing, or unchanging over a region, or calculate the values of x for which a function is increasing, decreasing, or unchanging.
		5.5	**Find** the stationary points of a function, where the gradient is zero.
		5.6	**Determine** if a stationary point is a 'Local Maximum', a 'Local Minimum' or a 'Point of Inflection' using the second derivative of the function.
		5.7	**Determine** if a stationary point is a 'Local Maximum', a 'Local Minimum' or a 'Point of Inflection' if the second derivative of the function $f''(a) = 0$, by finding $f'(a-h)$ and $f'(a+h)$.
		5.8	**Know and use** the derivatives of the functions e^x, $\ln x$, $\sin x$, $\cos x$, and $\tan x$.
		5.9	**Know and understand** how to find the derivatives of the sums/differences of constant multiples of functions.
		5.10	**Sketch** graphs of functions showing the position and nature of their stationary points.
		5.11	**Sketch** graphs of gradient functions showing the position and nature of their stationary points.
		5.12	**Solve** problems where the rate of change $\frac{\Delta y}{\Delta x}$ can be represented by the derivative $\frac{dy}{dx}$.
6	Understand integration.	6.1	**Know and use** the integrals of the functions $e^x, \frac{1}{x}$, $\sin x$, $\cos x$, and $\sec^2 x$.
		6.2	**Know and understand** how to find the integrals of the sums/differences of constant multiples of functions.
		6.3	**Know and understand** that the 'Definite Integral' calculates the integral between two limits and use the definite integral notation $\int_a^b f'(x)dx = [f(x)]_a^b = f(b) - f(a)$.
		6.4	**Know and understand** how to find the definite integral of functions with implicit and explicit limits.
		6.5	**Know and use** the fact that the definite integral of f with respect to x is the area between f, the x-axis, and the lines $x = a$ and $x = b$.
		6.6	**Know and understand** that areas underneath the x-axis are negative and must therefore be calculated separately.
		6.7	**Know and understand** how to find the areas between functions and the axis.
		6.8	**Know and understand** how to find the area between two functions using integration and/or geometry.

		6.9	**Use** the ‘Trapezium Rule’ to estimate the area under a function that cannot be integrated and determine whether the estimate is likely to be greater than or less than the true value. $\int_a^b y dx = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n)$ where $h = \frac{b-a}{n}$ and $y_i = f(a + ih)$.
7	Understand numerical methods.	7.1	**Estimate** the roots of a function by graphical means.
		7.2	**Estimate** the roots of a continuous function by locating a sign change for the function over an interval.
		7.3	**Estimate** the solutions of a function by ‘Iteration’.
		7.4	**Use** graphical methods to locate an initial value for which an ‘Iterative Formula’ will converge.
		7.5	**Know** that for the same function, different iterative formula will allow convergence on different roots, and that some iterative formula will diverge for certain initial values.

3	Pure Mathematics 3

Aim

Here we build on the foundations laid in Pure Mathematics 2. The aim of this subject content is to continue on with even more advanced algebraic techniques and calculus, while introducing vectors and complex numbers, and consolidating the ideas of mathematical proofs.

Note: There is the expectation that the content from Pure Mathematics 1 and 2 is known and it will be tested as part of this unit without being included below.

Learning Outcomes - The learner will:		**Assessment Criteria - The learner can:**	
1	Understand mathematical proof.	1.1	**Know and understand** the procedure and requirements for a mathematical proof.
		1.2	**Prove** an ‘Identity’ by algebraic manipulation, and use the correct symbol, ≡, to denote equivalence (“is always equal to”).
		1.3	**Prove** mathematical statements using ‘Proof by Deduction’.
		1.4	**Prove** mathematical statements using ‘Proof by Exhaustion’. {Less than 5 cases}
		1.5	**Disprove** mathematical statements using ‘Counter-Examples’.
		1.6	**Prove** mathematical statements using ‘Proof by Contradiction’, by proving that the ‘Negation’ of the statement is false.
2	Understand partial fractions.	2.1	**Know and understand** that ‘Improper Algebraic Fractions’ are fractions where the degree of the numerator is greater than or equal to the degree of the denominator.
		2.2	**Use** algebraic long division to convert an improper algebraic fraction into a mixed fraction: $\frac{f(x)}{g(x)} = q(x) + \frac{r}{g(x)}$
		2.3	**Convert** a proper fraction with linear factors in the denominator into partial fractions (maximum of three factors): $\frac{f(x)}{(x+a)(x+b)(x+c)} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{x+c}$
		2.4	**Convert** a proper fraction with linear factors, one of which is repeated, in the denominator into partial fractions (maximum of two factors): $\frac{f(x)}{(x+a)(x+b)^2} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{(x+b)^2}$
		2.5	**Convert** an improper fraction with linear factors in the denominator into partial fractions (maximum of two factors): $\frac{f(x)}{(x+b)(x+c)} = A(x) + \frac{B}{x+b} + \frac{C}{x+c}$ where $A(x)$ has degree equal to the degree of the numerator minus the degree of the denominator.
3	Understand differentiation.	3.1	**Differentiate** products of functions using the ‘Product Rule’, $\frac{d}{dx}f(x)g(x) = \left(\frac{d}{dx}f(x)\right) \times g(x) + f(x) \times \frac{d}{dx}g(x)$.

		3.2	**Differentiate** functions of functions using the 'Chain Rule', $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.
		3.3	**Apply** the special case of the chain rule, If $y = f(x)$ then $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$. {Students should be aware that this is not generally true and that $\frac{dy}{dx}$ is not a fraction.}
		3.4	**Differentiate** quotients of functions using the 'Quotient Rule', $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$, or by use of the chain and product rules. {The method will not be specified}
		3.5	**Use** the derivatives of the trigonometric functions sec, cot and cosec (given).
		3.6	**Differentiate** the inverse trigonometric functions arcsin, arccos and arctan.
		3.7	**Know and understand** how to find $\frac{dy}{dx}$ from the two polynomials $x = f(t)$ and $y = g(t)$ by 'Parametric Differentiation'.
		3.8	**Know and understand** how to find $\frac{dy}{dx}$ from polynomials in x and y by 'Implicit Differentiation'.
4	Understand integration.	4.1	**Use** standard integrals and the inverse of standard derivatives to integrate functions.
		4.2	**Use** the chain rule for differentiation to solve an integral by finding the function which differentiates to the integrand when the integrand is of the form $f(ax+b)$.
		4.3	**Use** trigonometric identities to integrate functions.
		4.4	**Solve** integrals of the form $\int \frac{f'(x)}{f(x)}dx$ by differentiating $\ln\|f(x)\|$.
		4.5	**Solve** integrals of the form $\int f'(x)\left(f(x)\right)^n dx$ by differentiating $\left(f(x)\right)^{n+1}$.
		4.6	**Solve** integrals using 'Integration by Parts', $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$. {<u>Only</u> when instructed to do so}
		4.7	**Solve** integrals using 'Integration by Substitution' by applying a given substitution.
		4.8	**Solve** integrals using partial fractions. {<u>Only</u> when instructed to do so}
5	Understand vectors.	5.1	**Know and understand** that a 'Scalar' has only a 'Magnitude' (size) and that a 'Vector' had magnitude and direction.
		5.2	**Recognise and use** standard notation for scalars and vector. {Scalar: S, s. Vector: $\overrightarrow{AB}$, **v**, $\vec{v}, \bar{v}$, or $\underline{v}$.}

		5.3	**Know and use** the fact that if $\overrightarrow{AB} = \overrightarrow{CD}$ then the line segments AB and CD are equal in length and parallel.
		5.4	**Know and use** the vector equivalences: $\overrightarrow{AB} = -\overrightarrow{BA}$ $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ $\overrightarrow{AB} + \overrightarrow{BA} = \mathbf{0}$ where $\mathbf{0}$ is the zero vector
		5.5	**Know and use** the 'Parallelogram Law' for addition of vectors.
		5.6	**Represent** vectors in 'Column Vector' form and perform scalar multiplication and vector addition in column vector form.
		5.7	**Represent** vectors in terms of multiples of 'Unit Vectors', using unit vector notation (e.g. $\hat{u}$), and perform scalar multiplication and vector addition in unit vector form.
		5.8	**Know and understand** how to find the magnitude of a vector, and use the magnitude notation, $\lvert\boldsymbol{v}\rvert$.
		5.9	**Represent** vectors in 'Magnitude-Direction' form with the angle between the vector and the positive x-axis. For $\boldsymbol{v} = a\hat{\imath} + b\hat{\jmath}$, $\lvert\boldsymbol{v}\rvert = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.
		5.10	**Solve** geometric problems in two dimensions.
		5.11	**Know and understand** that position vectors are vectors from the origin to a point in space, and how to represent vectors by the position vectors of their end points, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.
		5.12	**Know and understand** how to find the position vector of point that divides a line segment AB in the ratio $AP{:}PB = a{:}b$ using $\overrightarrow{OP} = \overrightarrow{OA} + \frac{a}{a+b}(\overrightarrow{OB} - \overrightarrow{OA})$
		5.13	**Know and use** the fact that if $\boldsymbol{a}$ and $\boldsymbol{b}$ are two non-parallel 2D vectors and $p\boldsymbol{a} + q\boldsymbol{b} = r\boldsymbol{a} + s\boldsymbol{b}$ then $p = r$ and $q = s$.
		5.14	**Know and understand** that vectors can have any number of dimensions.
		5.15	**Know and understand** that if $\boldsymbol{v} = (a, b, c)$ then $\lvert\boldsymbol{v}\rvert = \sqrt{a^2 + b^2 + c^2}$ and the angles with the positive axes are given by $\cos(\theta_x) = \frac{a}{\lvert\boldsymbol{v}\rvert}$, $\cos(\theta_y) = \frac{b}{\lvert\boldsymbol{v}\rvert}, and\ \cos(\theta_z) = \frac{c}{\lvert\boldsymbol{v}\rvert}$.
		5.16	**Know and understand** that the distance between two points, $(a_1, a_2, a_3, \ldots, a_n)$ and $(b_1, b_2, b_3, \ldots, b_n)$ is $\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + \cdots + (a_n - b_n)^2}$.
		5.17	**Know and use** the fact that if $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are vectors in 3D which do not lie in the same plane and $p\boldsymbol{a} + q\boldsymbol{b} + r\boldsymbol{c} = p'\boldsymbol{a} + q'\boldsymbol{b} + r'\boldsymbol{c}$ then $p = p'$, $q = q'$, and $r = r'$.
		5.18	**Know and use** the fact that the position vectors of every point on a straight line (the 'Vector Equation of the Straight Line') are given by $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$ where $\boldsymbol{a}$ is the position vector of a point on the line, $\boldsymbol{b}$ is a vector parallel to the line, and λ is a scalar.

		5.19	**Know and use** the fact that the vector equation of the straight line passing through the points with position vectors $\boldsymbol{c}$ and $\boldsymbol{d}$ is given by $\boldsymbol{r} = \boldsymbol{c} + \lambda(\boldsymbol{d} - \boldsymbol{c})$.
		5.20	**Know and understand** that the 'Scalar Product' of vectors is called the 'Dot Product', written $\boldsymbol{a}.\boldsymbol{b}$ ("$\boldsymbol{a}$ dot $\boldsymbol{b}$"), and is defined as $\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$ where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.
		5.21	**Know and use** the fact that $\boldsymbol{a}.\boldsymbol{b}$ is the magnitude of $\boldsymbol{a}$ in the direction of $\boldsymbol{b}$ and vice versa.
		5.22	**Know and use** the fact that the non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular if and only if $\boldsymbol{a}.\boldsymbol{b} = 0$.
		5.23	**Know and understand** that $\boldsymbol{a}.\boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}.\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$
		5.24	**Know and understand** how to find the 'Point of Intersection' of two non-parallel, non-skewed, lines in 3D.
		5.25	**Know and understand** how to find the angle between two intersecting straight lines in the directions $\boldsymbol{a}$ and $\boldsymbol{b}$ using $\cos\theta = \left\lvert\frac{\boldsymbol{a}.\boldsymbol{b}}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert}\right\rvert$.
6	Understand complex numbers.	6.1	**Know and understand** that the square root of -1 has no real solutions, and that the 'Imaginary Number' i is defined as $i = \sqrt{-1}$.
		6.2	**Know and understand** that imaginary numbers are of the form bi where b is a real number, and that 'Complex Numbers' are of the form $a + bi$ where a and b are real numbers.
		6.3	**Know and understand** that for the complex number $z = a + bi$, $\mathrm{Re}(z) = a$ is the real part of z, and $\mathrm{Im}(z) = b$ is the imaginary part of z.
		6.4	**Know and use** the fact that complex numbers are equal if and only if the real parts are equal and the imaginary parts are equal.
		6.5	**Add and subtract** complex numbers.
		6.6	**Multiply** complex numbers by real numbers.
		6.7	**Multiply** complex numbers of the form $z = a + bi$.
		6.8	**Know and use** the fact that the complex number $z = a + bi$ has a 'Complex Conjugate' $z^* = a - bi$.
		6.9	**Know and use** the fact that the sum of a complex number and its complex conjugate $z + z^* = 2a$ is real.
		6.10	**Know and use** the fact that the product of a complex number and its complex conjugate $zz^* = a^2 + b^2$ is real.
		6.11	**Divide** complex numbers of the form $\frac{a+bi}{c+di}$.

		6.12	**Know and understand** how to find the two distinct complex roots of a quadratic equation where $b^2 - 4ac < 0$.
		6.13	**Know and use** the fact that for a polynomial with real coefficients, $f(z)$, if the complex number z_1 is a root of $f(z) = 0$ then z_1^* is also a root.
		6.14	**Know and understand** how to find all the real and complex roots of cubic equations of the form $az^3 + bz^2 + cz + d = 0$, given one of the complex roots.
		6.15	**Know and understand** how to find all the real and complex roots of quartic equations of the form $az^4 + bz^3 + cz^2 + dz + e = 0$, given one of the complex roots.

4	**Mechanics 1**

Aim

Here we apply mathematical techniques to the study of static and moving objects. The aim of this subject content is to understand the effects of forces on objects, how they change their motion and transfer energy.

Note: There is the expectation that the content from Pure Mathematics 1 is known and it will be tested as part of this unit without being included below.

Learning Outcomes - The learner will:		Assessment Criteria - The learner can:	
0			‘Realistic’ objects are expected to be treated as particles with all forces acting **through** a single point. Forces are restricted to two dimensions, and motion to one dimension.
1	Understand statics.	1.1	**Know and understand** that ‘Forces’ are vectors and use vector notation to reference them.
		1.2	**Draw** ‘Free Body Diagrams’ of simple systems involving multiple forces.
		1.3	**Resolve** forces into orthogonal components (not just horizontally and vertically).
		1.4	**Calculate** the magnitude and direction of the ‘Resultant’ of parallel and orthogonal forces (resolution of a force may be a prerequisite).
		1.5	**Know and use** the fact that a particle in equilibrium is acted upon by a resultant force equal to zero, and that the vector sum of components in any direction is equal to zero.
		1.6	**Calculate** the magnitude and/or direction of the force required to keep a particle in equilibrium.
		1.7	**Know and use** Newton’s Third Law – When an object exerts a force on a second object, the second object exerts an equal and opposite force on the first.
		1.8	**Identify** Newton’s Third Law force pairs and use them to complete the forces acting on a system.
		1.9	**Know and understand** that a contact force between two objects can be represented by the ‘Normal’ component (resists motion through the surface) and the ‘Frictional’ component (resists motion along the surface).
		1.10	**Understand** the concept of ‘Limiting Equilibrium’ and know the language used to refer to situations where the equilibrium is on the limit.
		1.11	**Know and understand** that μ is the ‘Coefficient of Limiting Friction’ if the object is not moving, and how it applies to limiting and non-limiting equilibrium situations.
		1.12	**Solve** limiting equilibrium problems using $F = \mu R$ or $F \leq \mu R$ as appropriate.

2	Understand kinematics.	2.1	**Know and understand** that 'Displacement' is a vector quantity, and that 'Distance' is the scalar equivalent.
		2.2	**Know and understand** that 'Velocity' is a vector quantity that is the 'Rate of change of displacement', and that 'Speed' is the scalar equivalent.
		2.3	**Know and understand** that 'Acceleration' is a vector quantity that is the 'Rate of change of velocity'. can
		2.4	**Know and understand** that 'Deceleration' is the 'Rate of decrease of velocity'.
		2.5	**Solve** problems using $\text{Average velocity} = \frac{\text{Change in displacement}}{\text{Time taken}}$ and $\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$.
		2.6	**Solve** problems involving constant acceleration using the SUVAT equations: $s = ut + \frac{1}{2}at^2$, $v = u + at$, $v^2 = u^2 + 2as$, and $s = \frac{1}{2}(u + v)t$.
		2.7	**Know and use** the approximation for the acceleration due to gravity at the Earth's surface, $g = 9.81\ \mathrm{m/s}$. $\{g = 10\ \mathrm{m/s}$ is insufficient for calculations but may be used for estimations}.
		2.8	**Sketch** displacement-time and velocity-time graphs.
		2.9	**Know and understand** how to find the velocity at a point in time from the gradient of a displacement time graph at that point.
		2.10	**Know and understand** how to find the total displacement over a time from the area under a velocity-time graph over that time.
		2.11	**Find** the acceleration at a point in time from the gradient of a velocity-time graph at that point.
		2.12	**Know and understand** how to find differentiation and integration with respect to time (Pure 1 only) to solve problems involving the functional representation of displacement, velocity and/or acceleration.
		2.13	**Know and use** the fact that 'Linear Momentum' is a vector quantity that is the product of mass and velocity, $p = mv$.
		2.14	**Know and use** the fact that the 'Conservation of Linear Momentum' means that Total Momentum Before = Total Momentum After if no external force acts (e.g. friction)
		2.14	**Solve** problems involving collisions and explosions where the conservation of momentum applies.
3	Understand the action of resultant forces.	3.1	**Know and use** Newton's First Law – An object maintains its state of motion unless acted upon by an external resultant force.
		3.2	**Know and use** Newton's Second Law – An object accelerates (changes speed and/or direction) when acted upon by an external resultant force.

		3.3	**Know and understand** that Newton's 2nd law implies that Force is proportional to the rate of change of momentum.
		3.4	**Solve** problems using $F = ma$ and $F = \frac{d(mv)}{dt}$.
		3.5	**Know and understand** that 'Weight' is the force on a mass due to gravity and make use of the equation $W = mg$. {$g = 9.81\ \mathrm{m}/s$ at the Earth's surface}
		3.6	**Know and understand** that objects can be connected with 'Light Inextensible Strings' or 'Light Rigid Bars' as models of 'Perfect Connectors' (no mass, no change in form).
		3.7	**Know and understand** that 'Smooth' objects/surfaces have no friction.
		3.8	**Solve** problems where a particle moves under the action of constant forces, including up or down inclined planes. {Not including circular motion}
		3.9	**Solve** problems where the forces are not constant by considering a specific instant.
		3.10	**Solve** problems where two particles are connected by a perfect connector.
		3.11	**Know and understand** that an 'Impulse' is a change in momentum, $\mathrm{Impulse} = \Delta p = mv - mu$.
		3.12	**Calculate** the forces involved in collisions using $F = \frac{\mathrm{Impulse}}{\mathrm{Time\ Taken}}$.
		3.13	**Know and understand** that μ is the coefficient of dynamic friction if the object is moving.
		3.14	**Solve** limiting equilibrium problems using $F = \mu R$.
4	Understand energy, work and power.	4.1	**Know and understand** that the 'Work Done' is the energy transferred during a process, and that 'Energy' is the capacity to do work.
		4.2	**Know and understand** that the work done by a force is proportional to the product of the force and the distance the point on which force acts moves in the direction of the force.
		4.3	**Solve** problems where displacement is not in the direction of the force using $W = Fd\cos\theta$.

		4.4	**Know and use** the fact that the energy stored in a moving object is called 'Kinetic Energy', where $K.E.=\frac{1}{2}mv^2$.
		4.5	**Know and use** the fact that the energy stored in the gravitational field due to an object with mass is called 'Gravitational Potential Energy', where $G.P.E.=mgh$.
		4.6	**Know and use** the fact that the energy stored in a stretched/compressed spring is called 'Elastic Potential Energy', where $E.P.E=\frac{1}{2}kx^2$.
		4.7	**Know and use** the fact that the 'Conservation of Energy' means that $\text{Total Energy Before} = \text{Total Energy After}$ if no external force acts (e.g. friction) – i.e. no work is done on or by the system.
		4.8	**Know and understand** that the geometry of a particles motion is irrelevant if energy is conserved, only the initial and final configurations matter.
		4.9	**Solve** problems using the conservation of energy.
		4.10	**Know and understand** that 'Power' is the rate at which work is done.
		4.11	**Solve** problems using $P=\frac{W}{t}=Fv$.

5	Mechanics 2		
Aim Here we apply mathematical techniques to the study of more complex motion. The aim of this content subject is to understand two-dimensional motion and non-point-like objects. Note: There is the expectation that the content from Pure Mathematics 1 is known and it will be tested as part of this unit without being included below.			
Learning Outcomes - The learner will:		**Assessment Criteria - The learner can:**	
1	Understand projectile motion.	1.1	**Resolve** velocity vectors in to horizontal and vertical components.
		1.2	**Know and understand** that, in the absence of air resistance, horizontal acceleration is zero.
		1.3	**Know and understand** that, in the absence of air resistance, vertical acceleration is the acceleration due to gravity.
		1.4	**Know and understand** that the vertical and horizontal motions do not influence each other and can therefore be considered separately.
		1.5	**Apply** the SUVAT equations separately to the vertical and horizontal motions.
		1.6	**Solve** projectile motion problems where the projectile returns to the same or lower height than the one it is launched from.
		1.7	**Calculate** distances, speed and angles at specific points or times.
		1.8	**Calculate** the 'Time of Flight', the time between launch and the projectile reaching the horizontal plane.
		1.9	**Calculate** the 'Range', the distance travelled horizontally between launch and the projectile reaching the horizontal plane.
		1.10	**Calculate** the greatest height of the projectile, when the vertical velocity is equal to zero.
		1.11	**Describe** the effects on the motion of including air resistance or changing the acceleration due to gravity.
2	Understand forces on extended objects. (moments and centre of mass)	2.1	**Know and understand** that a 'Moment' is a turning force given by the magnitude of a force multiplied by the perpendicular distance to the pivot point.
		2.2	**Calculate** moments using $\text{Moment} = Fd$ or $\text{Moment} = Fd\sin\theta$.
		2.3	**Know and understand** that moments have a direction, 'Clockwise' or 'Anti-clockwise'.
		2.4	**Calculate** the 'Resultant Moment' by taking the sum of a set of moments.

		2.5	**Know and understand** how to find the magnitude of a force or its distance from the pivot such that a system is in equilibrium (resultant force and resultant moment are equal to zero).
		2.6	**Know and understand** that a 'Couple' is a pair of equal anti-parallel forces.
		2.7	**Prove** that the resultant moment of a couple is equal to the magnitude of one of the force multiplied by the distance between them regardless of the position of the centre of rotation.
		2.8	**Know and use** the fact that if there are multiple pivots then moments can be taken about any one of them.
		2.9	**Know and use** the fact that if a beam is on the point of rotating (tilting) about a pivot then the force on all other pivots is zero.
		2.10	**Know and understand** that the 'Centre of Mass' of an object is the point at which all the mass of the object appears to act.
		2.11	**Know and use** the fact that the centre of mass of a uniform bar is the mid-point of the bar.
		2.12	**Use** moments to find the location of the centre of mass of a non-uniform bar.
		2.13	**Know and understand** how to find the centre of mass of a set of particles on a straight line using $\sum m_i x_i = \bar{x}\sum m_i$ where $\bar{x}$ is the position of the centre of mass.
		2.14	**Prove** $\sum m_i x_i = \bar{x}\sum m_i$ for a set of particles on a straight line.
		2.15	**Know and understand** how to find the centre of mass of a set of points arranged in a plane by applying $\sum m_i x_i = \bar{x}\sum m_i$ to find the x-coordinate and $\sum m_i y_i = \bar{y}\sum m_i$ to find the y-coordinate (individually or using vector notation).
		2.16	**Know and understand** how to find the centre of mass of a simple uniform plane lamina (circle, rectangle, triangle).
		2.17	**Know and understand** how to find the centre of mass of a composite shape by the addition or subtraction of simple uniform plane laminas.
		2.18	**Solve** dynamic and equilibrium problems involving non-uniform bars and uniform plane laminas.
3	Understand circular motion and simple harmonic motion.	3.1	**Know and use** the fact that the 'Angular Velocity', ω, is defined as the number of radians turned through in one second (in the direction of rotation), and is given by $\omega = \frac{\theta}{t}$.
		3.2	**Know and use** the link between 'Linear Speed', v, and 'Angular Speed', ω, for motion in a circle of radius r, $v = \omega r$.

		3.3	**Know and understand** that circular motion has a 'Frequency', f, the number of complete revolutions per second, and a 'Period', T, the time taken in seconds for one complete revolution.
		3.4	**Know and use** the link between f, T, and ω, $f = \frac{1}{T} = \frac{\omega}{2\pi}$.
		3.5	**Show** that an object travelling in a circle at constant angular velocity is accelerating, and that the 'Centripetal Acceleration' is given by $a = \frac{v^2}{r} = r\omega^2$.
		3.6	**Use** Newton's laws to show that an object undergoing centripetal acceleration must be acted upon by a 'Centripetal Force' given by $F = \frac{mv^2}{r} = mr\omega^2$, and that both acceleration and force are directed towards the centre of the circle.
		3.7	**Calculate** the linear speed, angular speed, centripetal acceleration, centripetal force and/or radius of motion of an object moving in a circular path.
		3.8	**Show** that the x-coordinate of an object travelling in a circle about the origin is given by $x = R\cos(\omega t)$ where R is the radius of the circle. {Angle is in radians}
		3.9	**Show** by differentiation that the velocity and acceleration in the positive x-direction of an object travelling in a circle are given by $v = -R\omega\sin(\omega \mathrm{t})$, and $a = -R\omega^2\cos(\omega t)$. {Angles are in radians}
		3.10	**Know and understand** that the definition of 'Simple Harmonic Motion', 'SHM', is the motion of an object subject to the resultant force $F = -kx$ where k is a constant and x is the displacement from the equilibrium position along the positive x-axis.
		3.11	**Show** that SHM must obey the equations $x = A\cos(\omega t)$, $v = -A\omega\sin(\omega \mathrm{t})$, and $a = -A\omega^2\cos(\omega t)$, where A is the amplitude of oscillation and $\omega = \sqrt{\frac{k}{m}}$. Hence, show the link between SHM and circular motion. {Angles are in radians}

6	Probability and Statistics		
Aim Probability and statistics are the study of complex systems through averaging over large numbers of similar events. The aim of this subject content is to evaluate models of random events or data from experiments. **Note:** There is the expectation that the content from Pure Mathematics 1 is known and it will be tested as part of this unit without being included below.			
Learning Outcomes - The learner will:		**Assessment Criteria - The learner can:**	
1	Understand probability.	1.1	**Know and understand** that 'Probability' is a measure of the likelihood of a specific event happening, and that it expressed as a decimal or a percentage (0, 0%, Impossible -> 1, 100%, Certain).
		1.2	**Know and understand** the meaning of the terms 'Experiment', 'Event', 'Outcome', 'Sample Space', 'Mutually Exclusive', 'Independent', 'Equally Likely' and 'Biased' with regards to probability.
		1.3	**Know and use** the fact that the sum of the probabilities of all possible events equals one.
		1.4	**Use** P(event) notation to refer to the probability of an event.
		1.5	**Calculate** the probability of specific outcomes for simple dice rolling or card choosing experiments (one roll or draw).
		1.6	**Use** 'Sample Space Diagrams', 'Venn Diagrams', 'Tree Diagrams' and 'Histograms' to map out the probability space and retrieve probability data.
		1.7	**Know and understand** that the 'Intersection' of A and B refers to the outcome 'A **and** B', and is denoted by $A \cap B$.
		1.8	**Know and understand** that the 'Union' of A and B refers to the outcome 'A **or** B', and is denoted by $A \cup B$.
		1.9	**Know and understand** that the 'Complement' of A refers to the outcome '**not** A', and is denoted by A'.
		1.10	**Know and use** the fact that if two events are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.
		1.11	**Know and use** the fact that if two events are independent then $P(A \cap B) = P(A) \times P(B)$ and use the multiplication rule to determine if events are independent.
		1.12	**Know and use** the notation $P(A\|B)$ for A **given** B and $P(A\|B')$ for A **given** (**not** B) for 'Conditional Probability'.

		1.13	**Know and use** the fact that for independent events $P(A\|B) = P(A\|B') = P(A)$ and $P(B\|A) = P(B\|A') = P(B)$, and use these conditions to determine the independence of events.
		1.14	**Know and use** the 'Addition Formula' $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and prove it using Venn diagrams or otherwise.
		1.15	**Know and use** the 'Multiplication Formula' $P(A \cap B) = P(A\|B) \times P(B)$, and prove it using Venn diagrams or otherwise.
		1.16	**Calculate** the probabilities of outcomes of conditional and/or successive events.
2	Understand the representation and characterisation of data.	2.1	**Know and understand** the difference between 'Qualitative' and 'Quantitative', 'Discrete' and 'Continuous' data, and choose a reasonable representation for the data.
		2.2	**Draw** histograms, box plots and stem and leaf diagrams for data sets and extract information from them.
		2.3	**Use** 'Grouped Frequency Tables' to group data into 'Classes' and find the class boundaries, midpoint and width.
		2.4	**Know and understand** that 'Measures of Location' describe a position in a data set as a single value, and that 'Measures of Central Tendency' describe the centre of the data.
		2.5	**Calculate** the measures of central tendency for a set of data – 'Mean', 'Median' and 'Mode' (or 'Modal Class').
		2.6	**Calculate** the combined mean for sets – If set A has size n_a and mean $\overline{x_a}$, and set B has size n_b and mean $\overline{x_b}$, then the mean of A and B is $\bar{x} = \frac{n_a\overline{x_a}+n_b\overline{x_b}}{n_1+n_2}$.
		2.7	**Calculate** the mean from a frequency table using $\bar{x} = \frac{\sum xf}{\sum f}$.
		2.8	**Calculate** measures of location for a set of data – 'Quartiles' and 'Percentiles'.
		2.9	**Use** 'Interpolation' to estimate the median, quartiles and percentiles from a grouped frequency table.
		2.10	**Know and understand** how to find the quartiles from grouped continuous data or data presented in a cumulative frequency table.
		2.11	**Know and understand** that 'Measures of Spread' describe the distribution of the data as a single value.
		2.12	**Calculate** the simple measures of spread for a set of data – 'Range', 'Interquartile Range' and 'Interpercentile Range'.
		2.13	**Calculate** the 'Summary Statistic' $S_{xx} = \sum(x-\bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$.
		2.14	**Calculate** the 'Variance' of a set of data using $\sigma^2 = \frac{S_{xx}}{n}$. (measure of spread)

		2.15	**Calculate** the 'Standard Deviation', the square root of the variance, of a set of data using $\sigma = \sqrt{\frac{S_{xx}}{n}}$. (measure of spread)
		2.16	**Apply** coding of the form $y = (x - a)/b$ where *a* and *b* are given.
		2.16	**Calculate** the mean and standard deviation of coded data and use them to calculate the mean ($\bar{x} = b\bar{y} + a$) and standard deviation ($\sigma_x = b\sigma_y$) of the original data.
		2.17	**Use** a given definition for outliers, or impossibility arguments, to identify anomalies and 'Clean' the data.
		2.18	**Identify** whether a data set is 'Positively Skewed', 'Negatively Skewed' or 'Symmetrical' from frequency tables, bar charts or box plots (calculation not required).
		2.19	**Compare** two data sets using measures of location, measures of spread and skewedness.
3	Understand discrete random variables.	3.1	**Know and understand** that a 'Discrete Random Variable' is a variable that can only take specific numerical values, and the outcome cannot be known before the experiment is conducted (e.g. the value of a dice roll).
		3.2	**Know and understand** that 'Probability Distribution' for a discrete random variable is 'Uniform' if the probability of each outcome is the same (e.g. the value of a fair dice roll).
		3.3	**Use** the notation for random variables, where $P(X = x)$ is the probability that the random variable X has the outcome x.
		3.4	**Construct** tables and/or diagrams to display the probability distribution for a discrete random variable.
		3.5	**Calculate** the 'Cumulative Distribution Function' for the outcome of a discrete random variable, $F(x) = P(X \leq x)$.
		3.6	**Construct** cumulative distribution tables.
		3.7	**Calculate** the 'Expected Value' of a discrete random variable, $\mathrm{E}(X) = \sum xP(X = x)$.
		3.8	**Calculate** the 'Variance' of a discrete random variable, $\mathrm{Var}(X) = \mathrm{E}\left(\left(X - \mathrm{E}(X)\right)^2\right) = \mathrm{E}(X^2) - \left(E(X)\right)^2$.

		3.9	**Know and use** the fact that if X is a discrete random variable then the function $f(X)$ is also a discrete random variable.
		3.10	**Calculate** the expected value of a function $\mathrm{E}\big(f(X)\big) = \sum f(X)P(X = x)$.
		3.11	**Know and use** $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, and $\mathrm{E}(X + Y) = \mathrm{E}(X) + \mathrm{E}(Y)$.
		3.12	**Know and use** $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$.
		3.13	**Know and use** the fact that if the probability distribution is uniform and defined over the set [1,2,3,…,n] then $\mathrm{E}(X) = \frac{n+1}{2}$ and $\mathrm{Var}(X) = \frac{(n+1)(n-1)}{12}$.
4	Understand the normal distribution.	4.1	**Know and understand** that a 'Continuous Random Variable' can take any value (usually within a range), and that the probability that the variable takes a specific value is therefore equal to zero.
		4.2	**Know and use** the fact that the probability that a continuous random variable takes a value within a given range is given by the area under the distribution curve across that range, and that the area under the probability distribution curve is equal to one.
		4.3	**Know and use** the properties of the 'Normal Distribution': Determined uniquely by the 'Population Mean' μ and the 'Population Variance' σ^2 Symmetrical about the mean (mean = median = mode) Bell shaped with asymptotes at both ends and points of inflection at $\mu \pm \sigma$ Area under the curve = 1 Fixed percentages of the data lie within standard deviations of the mean $(\pm 1\sigma: 68\%, \pm 2\sigma: 95\%, \pm 3\sigma: 99.7\%)$
		4.4	**Know and understand** that a normally distributed variable, X, is denoted by $X \sim N(\mu, \sigma^2)$.
		4.5	**Know and use** the fact that the 'Standard Normal Distribution', $Z \sim N(0, 1^2)$, has a mean of zero and a variance of 1.
		4.6	**Use** coding to standardise a normal distribution such that $Z = \frac{X-\mu}{\sigma}$, or find unknown means or variances from a standardised distribution.
		4.7	**Use** labelled diagrams or data tables to determine probabilities for a normally distributed variable, and values of the variable for which $p = P(Z > z)$ given p.
5	Understand correlation and regression.	5.1	**Know and understand** that 'Bivariate' data is pairs of values of two variables.
		5.2	**Know and understand** that bivariate data is plotted on 'Scatter Diagrams' and extract information from them.

		5.3	**Know and understand** that bivariate data consists of an 'Independent (or Explanatory) Variable' whose values are chosen by the researcher and is always plotted on the horizontal axis, and a 'Dependent (or Response) Variable' that is measured by the researcher and is always plotted on the vertical axis.
		5.4	**Know and understand** that 'Correlation' is the linear relationship between the independent and the dependent variables and can be positive or negative, strong or weak, or zero.
		5.5	**Know and understand** that 'Linear Regression' fits a straight line ('Line of Best Fit') to the data set, the regression line of y on x is of the form $y = ax + b$.
		5.6	**Calculate** the bivariate summary statistics $S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$, $S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$, and $S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$.
		5.7	**Calculate** the 'Least Squares Linear Regression Line' for a data set using $a = S_{xy}/S_{xx}$ and $b = \bar{y} - a\bar{x}$, and use it to 'Interpolate' the dependent variable.
		5.8	**Know** that the 'Product Moment Correlation Coefficient'.
		5.9	**Calculate** the product moment correlation coefficient using $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.
		5.10	**Apply** coding to one or both variables, and know that the codings do not change the value of r.
		5.11	**Make** conclusions about bivariate data and use extrapolation to estimate values outside the data range.

APPENDIX

MATHEMATICAL FORMULAE AND IDENTITIES

Algebra

Complete the square: if $p(x+q)^2 + r = 0$ then $x = -q \pm \sqrt{-\frac{r}{p}}$.

Modulus: $|a| = |b| \iff a^2 = b^2$ and $|x-a| < b \iff a-b < x < a+b$

Arithmetic series:

$u_n = a + (n-1)d$ $\qquad$ $S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n(a+u_n)$

Geometric series:

$u_n = ar^{n-1}$ $\qquad$ $S_n = \frac{a(1-r^n)}{1-r}$ $(r \neq 1)$ $\qquad$ $S_\infty = \frac{a}{1-r}$ $(|r| < 1)$

Binomial Expansion: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Summations:

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ $\qquad$ $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ $\qquad$ $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

Geometry

Arc length of a circle: $s = r\theta$

Sector area of a circle: $A = \frac{1}{2}r^2\theta$

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Surface area of a cone: $A = \pi rl + \pi r^2$ where l is the length of the slanted side.

Volume of a cone or pyramid: $V = \frac{1}{3}\pi r^2 h$ where h is the height of the cone.

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Trigonometric Identities

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$
$\sin^2\theta + \cos^2\theta \equiv 1$
$\sec^2 \theta \equiv 1 + \tan^2 \theta$
$\mathrm{cosec}^2 \theta \equiv 1 + \cot^2 \theta$

$\sin A + \sin B \equiv 2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$
$\sin A - \sin B \equiv 2 \cos \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$
$\cos A + \cos B \equiv 2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$
$\cos A - \cos B \equiv 2 \sin \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$

$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ where $(A \pm B) \neq \left(n + \frac{1}{2}\right)\pi$

$\sin(2A) \equiv 2\sin A \cos A$
$\cos(2A) \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$
$\tan(2A) \equiv \frac{2\tan A}{1-\tan^2 A}$ where $(2A) \neq \left(n + \frac{1}{2}\right)\pi$

$a \sin x \pm b \cos x = R \sin (x \pm \alpha)$
$a \cos x \pm b \sin x = R \cos (x \mp \alpha)$
where $R > 0$, $0 < \alpha < \frac{\pi}{2}$, $R \cos \alpha = a$, $R \sin \alpha = b$, and $R = \sqrt{a^2 + b^2}$.

Differentiation

$f(x)$	$f'(x)$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arcsinh} x$	$\frac{1}{\sqrt{1+x^2}}$
$\operatorname{arccosh} x$	$\frac{1}{\sqrt{x^2-1}}$
$\operatorname{arctanh} x$	$\frac{1}{1-x^2}$

Product Rule: $\frac{d}{dx}f(x)g(x) = \left(\frac{d}{dx}f(x)\right) \times g(x) + f(x) \times \frac{d}{dx}g(x)$

Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

If $y = f(x)$ then $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

Integration

a is a positive constant.

$f(x)$	$\int f(x)\,dx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln(\cosh x)$
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\|\dfrac{x-a}{x+a}\right\| \quad (x>a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\|\dfrac{a+x}{a-x}\right\| \quad (\|x\|<a)$
$\dfrac{1}{\sqrt{x^2+a^2}}$	$\operatorname{arcsinh}\left(\dfrac{x}{a}\right)$ $\ln\left(x+\sqrt{x^2+a^2}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arccosh}\left(\dfrac{x}{a}\right)$ $\ln\left(x+\sqrt{x^2-a^2}\right) \quad (x>a)$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\|x\|<a)$

Trapezium Rule: $\int_a^b y\,dx = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n)$ where $h = \frac{b-a}{n}$ and
$y_i = f(a + ih)$

Mechanics

$$\text{Average velocity} = \frac{\text{Change in displacement}}{\text{Time taken}}$$

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Uniformly accelerated motion:

$s = ut + \frac{1}{2}at^2$
$v = u + at$
$v^2 = u^2 + 2as$
$s = \frac{1}{2}(u + v)t$

Forces:

$F = ma = \frac{d(mv)}{dt} = \frac{\text{Impulse}}{\text{Time Taken}}$
$F \leq \mu R$
$W = Fd\cos\theta$
$P = \frac{W}{t} = Fv$

Moments: $\text{Moment} = Fd\sin\theta$

Circular Motion:

$F = \frac{mv^2}{r} = mr\omega^2$
$a = \frac{v^2}{r} = r\omega^2$

Centres of mass of uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\frac{3}{8}r$ from centre

Hemispherical shell of radius r: $\frac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\frac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\frac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{3}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Probability and Statistics

Mean of set A and set B: $\bar{x} = \frac{n_a\overline{x_a}+n_b\overline{x_b}}{n_a+n_b}$ where n_i is the size of set i, and $\overline{x_i}$ is the mean of set i.

Mean from a frequency table: $\bar{x} = \frac{\sum xf}{\sum f}$ where f is the frequency of x.

Variance: $\sigma^2 = \frac{S_{xx}}{n}$

Standard deviation: $\sigma = \sqrt{\frac{S_{xx}}{n}}$

Summary Statistics:

$$S_{xx} = \sum(x-\bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$
$$S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$
$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$$

Least squares linear regression: Line of y on x is $y = ax + b$ where $a = S_{xy}/S_{xx}$ and $b = \bar{y} - a\bar{x}$.

Product moment correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

Coding: If $y = \frac{x-a}{b}$ then $\bar{x} = b\bar{y} + a$ and $\sigma_x = b\sigma_y$.

Expected value of a discrete random variable: $\mathrm{E}(X) = \sum xP(X = x)$

Variance of a discrete random variable: $\mathrm{Var}(X) = \mathrm{E}\left(\left(X - \mathrm{E}(X)\right)^2\right) = \mathrm{E}(X^2) - \left(E(X)\right)^2$

Normal Distribution $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

If Z has a normal distribution with mean 0 and variance 1, then for each value of z, the table below gives the value of $\Phi(z)$, where $\Phi(z) = P(Z \leqslant z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2}\, dt$.

For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.

											ADD								
z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table below gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291