LRN International AS and A Level: Mathematics (7138)

Candidate Name	
Candidate Number	
Centre Name	
Centre Number	

Paper 1: **Sample Paper**

(2 hours)

It is necessary to respond on this question paper. You must have a soft pencil (preferably of type B or HB), a clean eraser and a dark blue or black pen.

INSTRUCTIONS:

- You must write your name, candidate number, centre name and centre number in the designated spaces.
- Attempt all the questions using a dark blue or black pen.
- You may use a soft pencil for graphs.
- If working is needed for any question it must be shown below that question.
- Do not use correction fluid.
- Avoid writing on any bar codes.
- You are allowed to use a calculator if needed.

INFORMATION:

- This paper has a total of 100 marks.
- The number of marks assigned for every question or its parts is indicated within brackets [ ].
- Rough work must be completed on this question paper; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question

1. The equation $f(x) = 2x^2 - 5x - (k + 2) = 0$, where k is a constant, has no real roots. Find the set of possible values of k.

[4]

2. **(a)** Given that $x + 4, x$ and $x - 1$ are three consecutive terms of a geometric progression, calculate the value of x and the common ratio, r [6]

(b) Is the series convergent? Explain. [1]

3. **(a)** If $f(x) = x^4 + 4x^3 - 2x^2 - 17x + k$, find the value of k for which $(x - 2)$ is a factor of $f(x)$. [2]

(b) When k has this value, find another factor of $f(x)$, of the form $x + p$, where p is a constant. [2]

(c) Factorise $f(x)$ completely. [3]

4. The function $f(x)$ is defined by $f(x) = x^3 + 8$, for $-2 \leq x \leq 3$.

(a) Find $f^{-1}(x)$ and determine its domain. [4]

(b) State the range of $f^{-1}(x)$. [1]

(c) The function $g(x)$ is defined by

$$g(x) = \frac{x+1}{x-1}, x \neq 1.$$

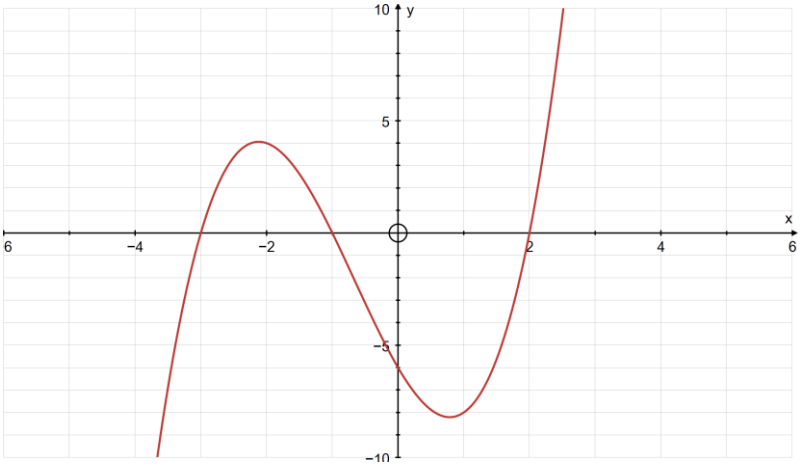
Show that $g(g(x)) = x$. [3]

(d) Write down $g^{-1}(x)$. [1]

5. The diagram below shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = x^3 + 2x^2 - x - 6.$$

The curve passes through the points $(-3, 0), (-1, 0), (0, -6)$ and $(2,0)$.



On separate diagrams, sketch the curve with equation

(a) $y = f(x + 3)$ [3]

(b) $y = f\left(\frac{1}{2}x\right)$ [3]

(c) $y = -f(x)$ [3]

Show, on each sketch, the coordinates of each point at which the graph meets the **both coordinate axes**.

6. Write down and simplify the first three terms of the expansion, in ascending powers of x, of:

(a) $(2+x)^5$ [3]

(b) $\left(1-\frac{x}{3}\right)^5$ [2]

(c) Hence, or otherwise, obtain the coefficient of x^2 in the expansion of

$$\left(2+\frac{x}{3}-\frac{x^2}{3}\right)^5$$

[4]

7. The circle C has equation $x^2 - 6x + y^2 - 14y + 33 = 0$.

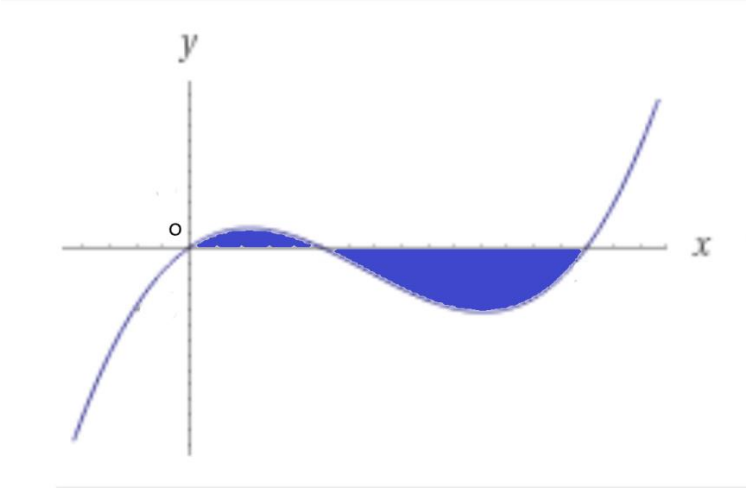
(a) Find the coordinates of the centre of C and the exact radius of C. [4]

(b) The line with equation $y = 3 - 2x$ intersects with C at the points A and B. Find the length of the chord AB, giving your answer in the form $k\sqrt{5}$. [8]

8. **(a)** Show that $sin\theta\ tan\theta\ + cos\theta\ = 7$ can be written as $sec\theta\ = 7$. [5]

(b) Hence solve $sin2x\ tan2x + cos2x = 7$ for $0 \leq x \leq \pi$, giving your answers to 3 decimal places. [5]

9.



The diagram shows the curve with equation $y = x^3 - 4x^2 + 3x$.

(a) Find the co-ordinates of the points where the curve crosses the x-axis. [3]

(b) Show that the total area of the shaded regions enclosed by the curve and the x-axis is $\frac{37}{12}$. [6]

(c) Find the x -coordinates of the turning points of the curve. [4]

10. Given that $\log_3 x = a$ and $\log_3 y = b$, find in terms of a and b,

(a) $\log_3(\frac{x^3}{y^2})$ [2]

(b) $\log_3(27x^2y)$ [2]

(c) It is given that $\log_3(\frac{x^3}{y^2})$= 4 and that $\log_3(27x^2y) = 8$. [3]

By forming simultaneous equations in a and b, show that $a = 2$ and find the value of b.

(d) Using the value of a and b, or otherwise, find the value of x and y. [2]

11.

$$f(x) = 2e^x - x^2 + 2x - 6 \ , x \in \mathcal{R} .$$

(a) Show that there is a root α of $f(x) = 0$ in the interval $[0.9, 1.1]$. [4]

(b) Show that the equation $f(x) = 0$ can be written in the form [3]

$$x = \ln\left(\frac{x^2-2x+6}{2}\right).$$

(c) Use the iterative formula $x_{n+1} = \ln\left(\frac{x^2{}_n - 2x_n + 6}{2}\right)$, $x_0 = 0.9$, to calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places. [4]