LRN International A Level: Mathematics (7138)

Candidate Name	
Candidate Number	
Centre Name	
Centre Number	

Paper 3: Mathematics **Sample Paper**

(2 hours)

It is necessary to respond on this question paper. You must have a soft pencil (preferably of type B or HB), a clean eraser and a dark blue or black pen.

INSTRUCTIONS:

- You must write your name, candidate number, centre name and centre number in the designated spaces.
- Attempt all the questions using a dark blue or black pen.
- You may use a soft pencil for graphs.
- If working is needed for any question it must be shown below that question.
- Do not use correction fluid.
- Avoid writing on any bar codes.
- You are allowed to use a calculator if needed.

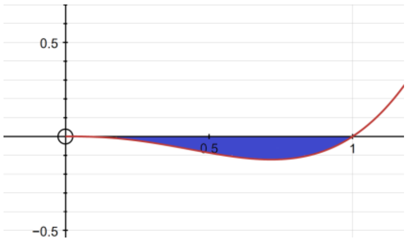
INFORMATION:

- This paper has a total of 100 marks.
- The number of marks assigned for every question or its parts is indicated within brackets [ ].
- Rough work must be completed on this question paper; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question

1. a) Use the quotient rule to show that $\frac{d}{dx}(tanx) = \sec^2 x$. [5]

(b) Use the product rule to show that $\frac{d}{dx}\left(e^{4x}\ln(4x)\right) = e^{4x}\left(\frac{1}{x} + 4\ln(4x)\right)$ [5]

2. (a) Use Integration by Parts to integrate $\int x^3 lnx\,dx$. [6]



b) The figure above shows the curve $y = x^3 \ln x$. Show that the area of the shaded region is $\frac{1}{16}$. [3]

3. **A curve C is described by the equation**

$$y^3 + 4x^2 - 2xy - e^x = 7$$

(a) Show that the point $(0, 2)$ lies on this curve. [2]

(b) Find an equation of the normal to C at the point $(0, 2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [11]

4. (a) Express $\frac{x-2}{(x-1)(2x-3)}$ in partial fractions. [4]

(b) Hence show that $\int \frac{x-2}{(x-1)(2x-3)}\,dx = \ln\left(\frac{k(x-1)}{\sqrt{2x-3}}\right)$, where k is a constant. [6]

5. Relative to a fixed origin O, the point A has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, the point B has position vector $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$, and the point C has position vector $\mathbf{i} - 2\mathbf{j}$.

(a) Find the cosine of angle ABC , leaving your answer in the form $k\sqrt{7}$, where $k \in \mathbb{Q}$. [7]

(b) Find the area of triangle ABC, giving your answer to 2 decimal places. [4]

6. (a) $3\cos\theta - sin\theta$ **in the form** $Rcos(\theta + \alpha)$, **where** R **and** α **are constants,** $R > 0$ **and** $0 < \alpha < 90°$ **and give the value of** α **to 2 decimal places.** [4]

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{3}{3\cos\theta - sin\theta - 2} = 12,$$

Giving your answers to two decimal places. [6]

7. (a) It is asserted that $|2x+3| \leq 7 \Rightarrow |x| \leq 2$.

Disprove this assertion by a counter-example. [3]

b) Use proof by contradiction to show that there exists no integers x and y for which

$$\mathbf{25x + 15y = 1.}$$

[5]

8.

$$z = 3 - 2i$$

(a) Show that $|z| = \sqrt{13}$ [2]

(b) Find, showing your working, the value of $|z^2|$. [2]

(c) Write down z^* **the conjugate of** z, **and find** $z + z^*$. [2]

(d) Show that $zz^* = |z|^2$ [2]

9. (a) Use the substitution $u = sinx$ **to evaluate** $\int_0^{\frac{\pi}{2}} \frac{cosx}{4+sinx}\,dx$**, giving the exact value.** [6]

(b)

$$y = \frac{cosx}{4+sinx}, 0 \leq x \leq \frac{\pi}{2}$$

Complete the table below, by giving the value of y when $x = \frac{\pi}{10}$, to 4 decimal places.

[1]

x	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$	$\frac{3\pi}{10}$	$\frac{2\pi}{5}$	$\frac{\pi}{2}$
y	0.25		0.1763	0.1222	0.0624	0

(c) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^{\frac{\pi}{2}} \frac{cosx}{4+sinx}\, dx,$$

giving your answer to 3 decimal places. [3]

(10) The curve C has parametric equations

$$x = csc\, 2\theta + 8,\ y = 2cot2\theta, \quad 0 \le \theta < 2\pi$$

(a) Show that $\frac{dy}{dx} = sec2\theta.$ [8]

(b) Find a cartesian equation of the curve. [3]